Equations Différentielles Ordinaire

Exercice N° 1:

```
import numpy as np

import matplotlib.pyplot as plt

from scipy import integrate


def euler1d(f,y0,t0,tN,N):

    t=np.linspace(t0,tN, N)

    y=np.zeros(N)

    y[0]=y0

    h=(t[1]-t[0])

    for i in range(1, len(t)):

        y[i]=y[i-1]+h*f(y[i-1], t[i-1])

    return y


def cauchy1d(y,t):

    return y

def comparaison1D(f, y0, t0, tN, N):

    t=np.linspace(t0,tN, N)

    yEuler = euler1d(f,y0,t0,tN,N)

    yAnalytic = y0*np.exp(t)

    R = integrate.odeint(f,y0,t)

    yOdeint = R[:, 0]


    plt.plot(t, yAnalytic)

    plt.plot(t, yEuler)

    plt.plot(t, yOdeint)

    plt.legend(("Analytic", "Euler", "Odeint"))

    plt.show()


    err1 = max(np.abs(yEuler-yAnalytic))

    err2 = max(np.abs(yOdeint-yAnalytic))


    return err1, err2

# Pour tester

print(comparaison1D(cauchy1d, 1,0, 10, 1000))
```

Exercice N° 2 :

On s'intéresse maintenant à une équation d'ordre 2, celle du pendule sans frottements :

$$\ddot{\theta} + \omega_0^2 \sin\theta = 0 \ avec\ \theta(t_0) = \theta_0\ et\ \dot{\theta}(t_0) = \dot{\theta}_0$$

On pose : $\dot{\theta} = v$

Ce qui donne : $\begin{cases} \dot{\theta} = v \\ \dot{v} = -\omega_0^2 \sin\theta \\ \theta(t_0) = \theta_0 \\ v(t_0) = \dot{\theta}_0 \end{cases}$

On utilise la notation vectorielle pour formuler le problème de Cauchy :

$X = \binom{\theta}{v}$ et $\dot{X} = \binom{v}{-\omega_0^2 \sin\theta}$

```
omega = 2*np.pi

T=2*np.pi/omega


def cauchy2d(X,t):

    theta = X[0]

    v= X[1]

    thetaprim = X[1]

    vprim=-(omega**2)*np.sin(theta)

    return np.array([thetaprim, vprim])

def euler2d(f,y0,t0,tN,N):

    t=np.linspace(t0,tN, N)

    y=np.zeros((N,2))

    y[0]=y0

    h=(t[1]-t[0])

    for i in range(1, len(t)):

        y[i]=y[i-1]+h*f(y[i-1], t[i-1])

    return y[:, 0]


def comparaison2D(f, y0, t0, tN, N):

    t=np.linspace(t0,tN, N)

    yEuler = euler2d(f,y0,t0,tN,N)

    R = integrate.odeint(f,y0,t)

    yOdeint = R[:, 0]

    plt.plot(t, yEuler)

    plt.plot(t, yOdeint)

    plt.legend(("Euler", "Odeint"))

    plt.show()

# Pour tester

comparaison2D(cauchy2d,[np.pi/6, 0],0, 5*T, 10000)
```